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WE CAN BUILD WHATEVER WE LIKE…CAN’T WE?

Wanda LEWIS*

* School of Engineering
University of Warwick
Coventry CV4 7AL, UK

E-mail: w.j.lewis@warwick.ac.uk URL: https://warwick.ac.uk/fac/sci/eng/people/wjl

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ABSTRACT

Introduction

The shape of our built environment is largely determined by architectural vision. Such a vision is often governed by subjective aesthetics and the need to make an impact; structural efficiency, optimality, and long-term durability are not a priority, and we are often left with non-optimal structures as a result of an architectural ‘whim’. Such structures are often branded as ‘innovative’. Examples include: Sydney Opera House (Fig. 1) [1], [2], the ‘Bird Nest’ stadium in China (Fig. 2), the London Millennium Dome and London Millennium bridge (Figs. 3 and 4) [3], and many others.

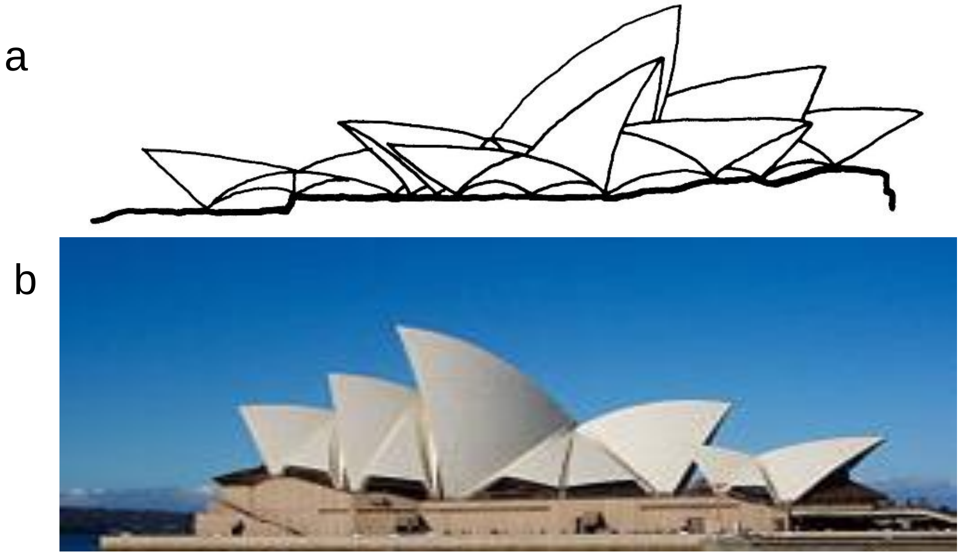


Fig. 1. Sydney Opera House 1957-1973; a) the original sketch lacking structural understanding, b) the final solution: a simplified structure costing AU$102 M (14 times the original estimate); poor acoustics and lighting; inadequate storage for instruments; shells require continuous cooling to an optimum 22.5° C

Fig. 2. Bejing National Stadium ‘Bird Nest’ 2003 – 2008. Artistically inspired by crackle-glazed Bejing pottery and heavily veined Chinese scholar stones. Deconstructivism style; cost: \$345 M - \$423 M; 420,000t of steel and 110,000t of concrete used, giving an estimated total of CO_2: 800,000t (5 times that of the Wembley Stadium, London, of similar capacity)

Fig. 3. London Millennium Dome 2000. Teflon-coated glass-fibre membrane on pre-stressed cables supported by masts via stay cables; an unnatural choice of form for a surface-stressed structure, huge forces present at the top of the membrane, stay cables proved too short to mate with the surface during construction (as the membrane crept towards its minimal form), saddling of the surface not eliminated

Even in situations where the intention of the architect was to produce an optimal structure, as in the case of the Gateway Arch in St. Louis (Fig. 5), the last minute change to the intended geometry (determined from various form-finding experiments), resulted in a non-optimal shape [4].

Fig. 4. London Millennium Bridge 2000. Architecture**:** Norman Foster, Engineering**:** Buro Happold; light aluminium deck prone to lateral vibrations. nearly horizontal suspension cables generated extremely high tension forces (22500kN) as in a suspension road bridge, equal length cables acted as guitar strings encouraging vibration, structure retro-fitted with dampers (at an additional cost of £5 M) to eliminate lateral sway

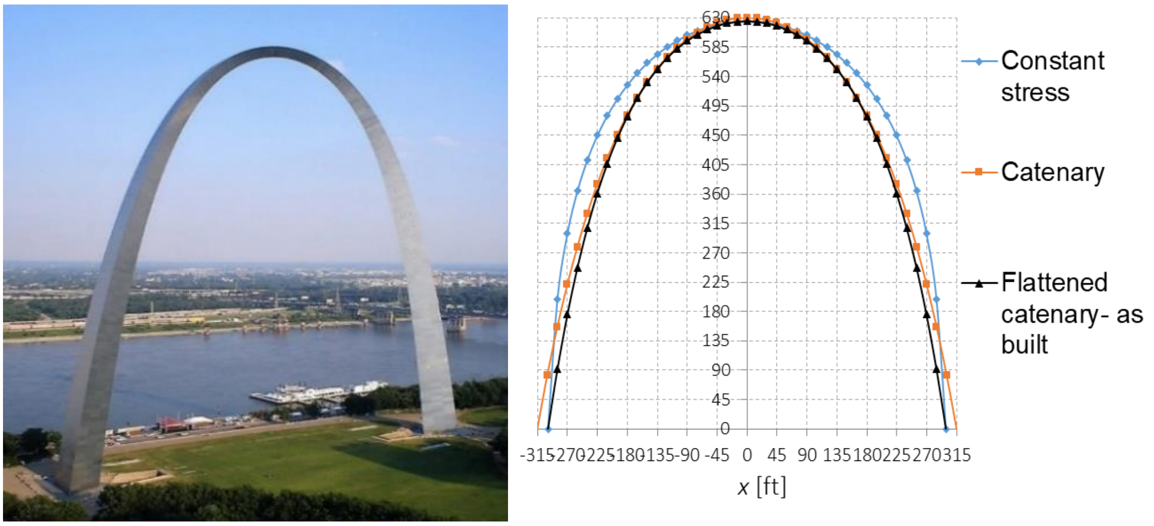
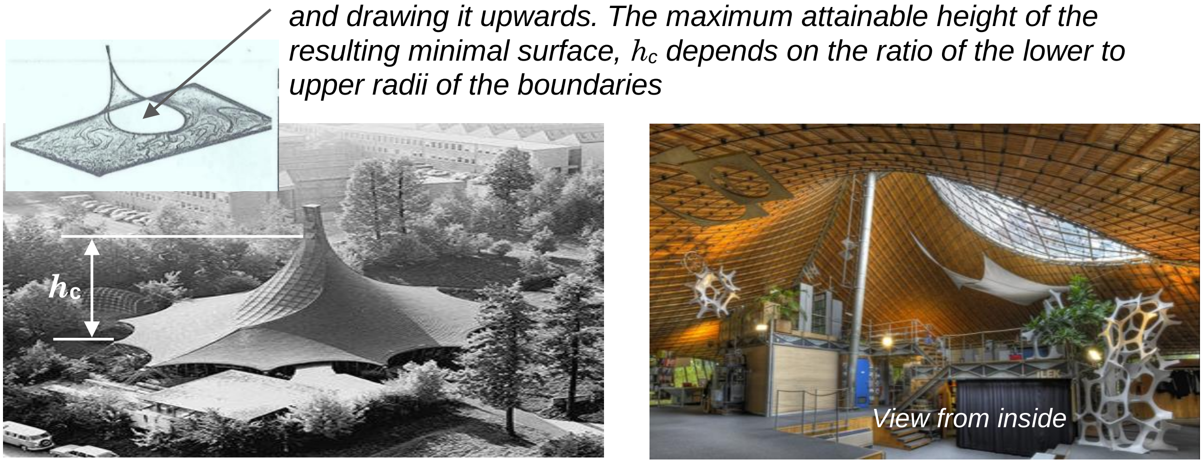


Fig. 5. Gateway Arch in St Louis. Centre-line profile officially defined as catenary but shape adjusted ‘by eye’ to that of a ‘flattened catenary’ [5]. The resulting structure is not an optimal, constant stress form (discussed later)

Climate change and concerns over sustainability/durability issues brought calls for improved architectural and engineering approaches to design. In the UK, for example, we use 2.5 times more material than the sustainable amount of resources (Whitelaw [6]). It is suggested here that a form-finding approach based on biomimicry, which implements principles observed in the formation of natural objects, such as bones, trees, shells, should be adopted at the conceptual design stage (whenever possible), as this can provide answers to problems of durability/sustainability facing our future built environment.

What is form-finding?

It is a process of shaping structures using, or controlling, forces developing in them. Form-finding has been proven to deliver optimal designs, in terms of material economy, durability and aesthetics, provided it is based on natural principles. Many techniques are possible, depending on the type of structure being modelled [7]. The technique presented here implements the principle of *constant stress*, present in soap-films. For example, roof-type structures (Figs. 6 and 7) have been modelled using a soap-film analogy.



Central window created by placing a string on a soap-film surface and drawing it upwards. The maximum attainable height of the resulting minimal surface, h_c depends on the ratio of the lower to upper radii of the boundaries

Fig. 6. IL Tent, Stuttgart 1965; a listed building. Design: Frei Otto/ Atelier Warmbronn utilised cable net and soap-film models

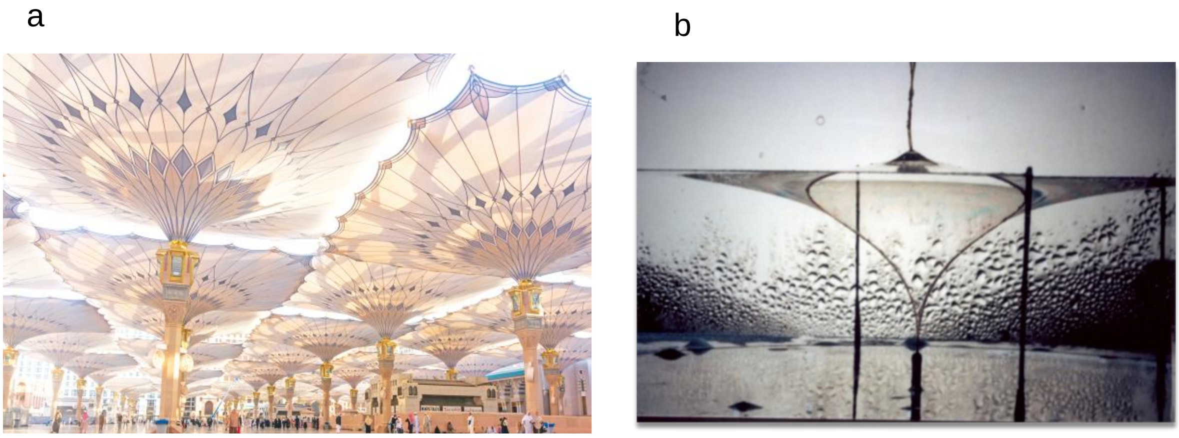


Fig. 7. 26 m x 26 m Convertible Umbrellas in Madinah, Saudi Arabia 2011. Design: SL-Rasch: a) umbrellas in fully open state, b) soap-film model used in the conceptual design

Constant stress principle in modelling moment-less arch structures

Latest research [8], involving an analytical form-finding approach, indicates that the *constant stress* principle can be used successfully in shaping efficient rigid structural forms, such as moment-less arches. These arches have the simplest stress response to loading, similar to natural objects. The approach is a form of reverse engineering in which the shape of the arch is a function of chosen span, span/rise and statistically prevalent load, such as the self-weight. The arches develop constant axial stress under this load, allowing the structure to work in its optimum state most of the time. Checking the structure for the ultimate load, involving permanent and variable loads, is necessary, to check that the design strength of the material is not exceeded. Should this occur, the value of the constant stress can be reduced, and the geometry of the structure re-calculated.

Figure 8 shows examples of 1:20 scaled models of constant stress, moment-less arches whose configuration and material distribution are a function of constant axial stress and applied loading. With reference to the data given below, it can be seen that the cross-section of the arch varies much more rapidly in the case of the high-rise arch (case a)). As reported in [8], the profiles of the constant stress arches are very similar to those of conventional, parabolic forms, but the former still use significantly less material (Fig. 9) and have much lower stress response to loading.

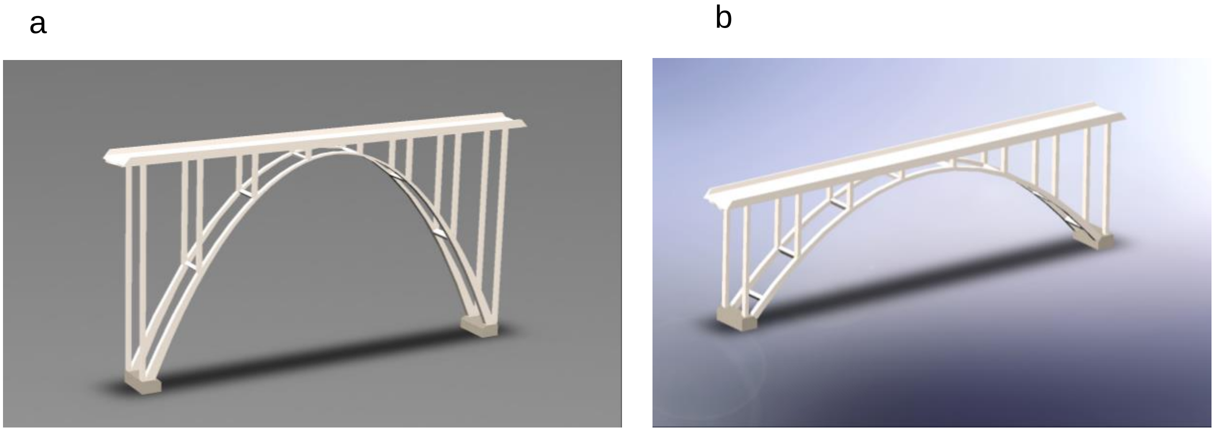


Fig. 8. 1:20 Models of moment-less arches of constant axial stress. Full-scale data: span l= 50 m, deck load w = 50 kN/m, arch load density q = 25 kN/m^3, stress f = 2.4 MPa. Case a): rise h = 25 m, l/h = 2, cross-sections: A_{top} = 0.3313 m^2, A_{bot}= 0.7985 m^2. Case b): rise h = 12.5 m, l/h=4, Cross-sections: A_{top} = 0.7473 m^2, A_{bot} = 1.089 m^2

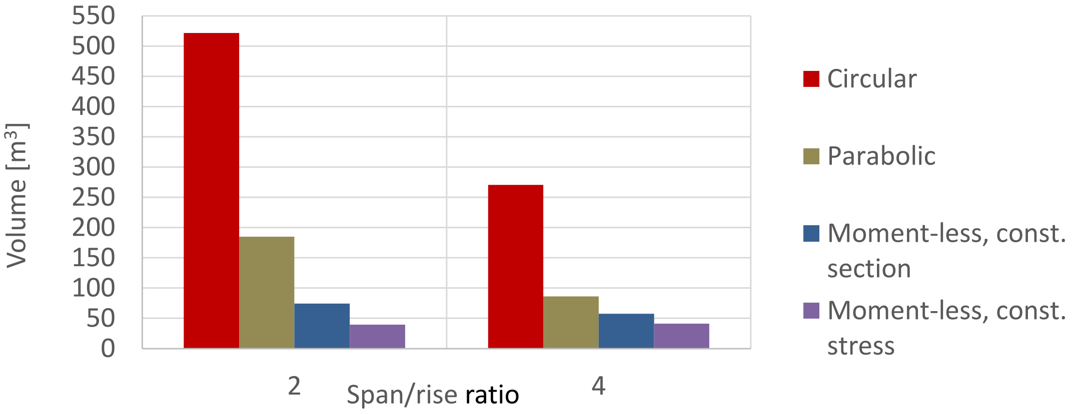


Fig. 9. Volume of material used by constant stress and other forms of arches

Comparisons presented above relate to another type of moment-less arch form, that of constant cross-section [4], as well as the dictated forms, which are of parabolic and circular configurations. All arches carry the same deck weight.

Existence of moment-less arches of constant axial stress

A unique finding of the research is that *the* ***shape of constant stress arches cannot be found for any combination of the input data***. Their existence depends on two dimensionless parameters: l/h and $ql/2f$, which define their design space [8]. In the case of free-standing, constant stress arches, l/h and f cannot be chosen independently.

Existence of constant surface stress membranes

Interestingly, membranes of constant surface stress, as exemplified by soap-films, also have limits imposed on their existence. For example, in the case of a catenoid (Fig. 10) the maximum ratio of the radius to the height of the structure must lie below 1.325; otherwise, the surface breaks into two disc-type surfaces [9]. In the case of unequal size rings, the maximum attainable height depends on the ratio of R_2/R_1 (Fig.11). Similar constraints apply to the IL Tent shown in Fig.6, reducing the height of the structure to a limit depending on the ratio of the radii of the boundary rings [10].

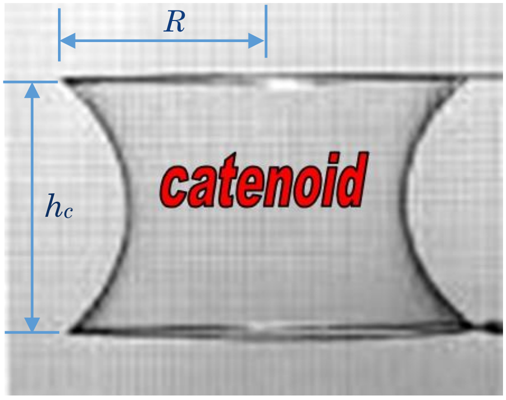


Fig. 10. Catenoid – a stable minimal surface (constant surface stress structure) modelled by a soap-film. The surface breaks up when $h_c / R > 1.325$ (for equal size rings)

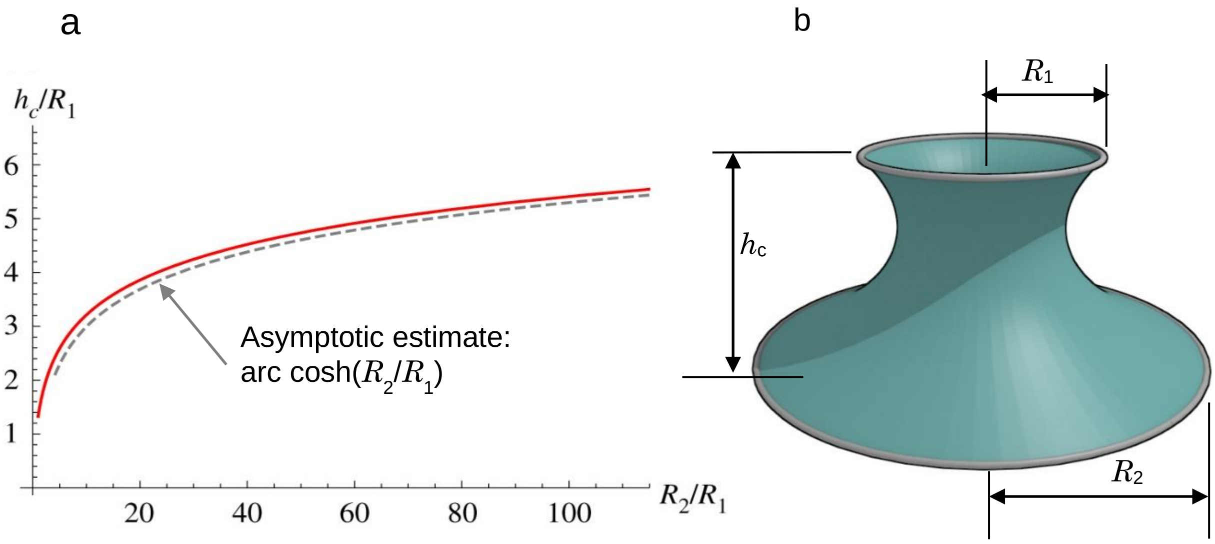


Fig. 11. Catenoid formed between unequal size rings: (a) maximum attainable height of the catenoid, h_c, as a function of R_2 / R_1, b) catenoid surface

Summary

- In the case of surface stressed membranes, the principle of constant surface stress translates to a minimum surface area
- In the case of arch structures, the principle of constant axial stress produces a minimum volume.
- Constant stress structures have a limited design space, but this space is very large; large enough to accommodate an enormous number of possible natural structural forms [11]. Their main feature of minimal stress response to loading provides optimal design solutions, in terms of material efficiency, aesthetics and durability.

Final remarks

The work presented here challenges established design approaches. Taking nature as the best teacher of optimum design leads us to reflect on the belief: *“we can built whatever we like… can’t we?”*

Acknowledgement

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